

Connecting the Concepts

An equation contains an equals sign; an expression does not. Be careful not to confuse simplifying an expression with solving an equation. When expressions are simplified, the result is an equivalent expression. When equations are solved, the result is a solution. Compare the following.

Simplify: $\frac{x-1}{6x} + \frac{4}{9}$.

SOLUTION We have

$$\begin{aligned}\frac{x-1}{6x} + \frac{4}{9} &= \frac{x-1}{6x} \cdot \frac{3}{3} + \frac{4}{9} \cdot \frac{2x}{2x} \\ &= \frac{3x-3}{18x} + \frac{8x}{18x} \\ &= \frac{11x-3}{18x}.\end{aligned}$$

The expressions

$$\frac{x-1}{6x} + \frac{4}{9} \quad \text{and} \quad \frac{11x-3}{18x}$$

are equivalent.

The equals signs indicate that all the expressions are equivalent.

Writing with the LCD, $18x$

The result is an expression.

Solve: $\frac{x-1}{6x} = \frac{4}{9}$.

SOLUTION We have

$$\begin{aligned}\frac{x-1}{6x} &= \frac{4}{9} \\ 18x \cdot \frac{x-1}{6x} &= 18x \cdot \frac{4}{9} \\ 3 \cdot \cancel{6x} \cdot \frac{x-1}{\cancel{6x}} &= 2 \cdot 9 \cdot x \cdot \frac{4}{9} \\ 3(x-1) &= 2x \cdot 4 \\ 3x-3 &= 8x \\ -3 &= 5x \\ -\frac{3}{5} &= x.\end{aligned}$$

Each line is an equivalent equation.

Multiplying by the LCM, $18x$

The result is a solution.

The solution of $\frac{x-1}{6x} = \frac{4}{9}$ is $-\frac{3}{5}$.

6.4 Exercise Set

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PRACTICE

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REVIEW

Concept Reinforcement Classify each of the following as either an expression or an equation.

1. $\frac{5x}{x+2} - \frac{3}{x} = 7$ Equation

2. $\frac{3}{x-4} + \frac{2}{x+4}$ Expression

3. $\frac{4}{t^2-1} + \frac{3}{t+1}$ Expression

4. $\frac{2}{t^2-1} + \frac{3}{t+1} = 5$ Equation

5. $\frac{2}{x+7} + \frac{6}{5x} = 4$ Equation

6. $\frac{5}{2x} - \frac{3}{x^2} = 7$ Equation

7. $\frac{t+3}{t-4} = \frac{t-5}{t-7}$ Equation

8. $\frac{7t}{2t-3} \div \frac{3t}{2t+3}$ Expression

9. $\frac{5x}{x^2-4} \cdot \frac{7}{x^2-5x+4}$ Expression

10. $\frac{7x}{2-x} = \frac{3}{4-x}$ Equation

Solve. If no solution exists, state this.

11. $\frac{3}{5} - \frac{2}{3} = \frac{x}{6} - \frac{2}{5}$

12. $\frac{5}{8} - \frac{3}{5} = \frac{x}{10} - \frac{1}{4}$

13. $\frac{1}{8} + \frac{1}{12} = \frac{1}{t} - \frac{24}{5}$

14. $\frac{1}{6} + \frac{1}{10} = \frac{1}{t} - \frac{15}{4}$

15. $\frac{x}{6} - \frac{6}{x} = 0$ $-6, 6$

16. $\frac{x}{7} - \frac{7}{x} = 0$ $-7, 7$

17. $\frac{2}{3} - \frac{1}{t} = \frac{7}{3t}$ 5

18. $\frac{1}{2} - \frac{2}{t} = \frac{3}{2t}$ 7

19. $\frac{n+2}{n-6} = \frac{1}{2}$ -10

20. $\frac{a-4}{a+6} = \frac{1}{3}$ 9

21. $\frac{12}{x} = \frac{x}{3}$ $-6, 6$

22. $\frac{x}{2} = \frac{18}{x}$ $-6, 6$

- Aha!** 23. $\frac{2}{6} + \frac{1}{2x} = \frac{1}{3}$ 24. $\frac{12}{15} - \frac{1}{3x} = \frac{4}{5}$
No solution No solution
25. $y + \frac{4}{y} = -5$ -4, -1 26. $t + \frac{6}{t} = -5$ -3, -2
27. $x - \frac{12}{x} = 4$ -2, 6 28. $y - \frac{14}{y} = 5$ -2, 7
29. $\frac{y+3}{y-3} = \frac{6}{y-3}$ 30. $\frac{4}{8-a} = \frac{4-a}{a-8}$
No solution No solution
- 31.** $\frac{x}{x-5} = \frac{25}{x^2-5x}$ -5 32. $\frac{t}{t-6} = \frac{36}{t^2-6t}$ -6
- 33.** $\frac{n+1}{n+2} = \frac{n-3}{n+1}$ - $\frac{7}{3}$ 34. $\frac{n+2}{n-3} = \frac{n+1}{n-2}$ $\frac{1}{2}$
- Aha!** 35. $\frac{x^2+4}{x-1} = \frac{5}{x-1}$ -1 36. $\frac{x^2-1}{x+2} = \frac{3}{x+2}$ 2
37. $\frac{6}{a+1} = \frac{a}{a-1}$ 2, 3 38. $\frac{4}{a-7} = \frac{-2a}{a+3}$ 2, 3
39. $\frac{60}{t-5} - \frac{18}{t} = \frac{40}{t}$ -145 40. $\frac{50}{t-2} - \frac{16}{t} = \frac{30}{t}$ -23
41. $\frac{3}{x-3} + \frac{5}{x+2} = \frac{5x}{x^2-x-6}$ No solution
42. $\frac{2}{x-2} + \frac{1}{x+4} = \frac{x}{x^2+2x-8}$ -3
- 43.** $\frac{3}{x} + \frac{x}{x+2} = \frac{4}{x^2+2x}$ -1
44. $\frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2+x}$ -4
45. $\frac{5}{x+2} - \frac{3}{x-2} = \frac{2x}{4-x^2}$ 4
46. $\frac{y+3}{y+2} - \frac{y}{y^2-4} = \frac{y}{y-2}$ -3
47. $\frac{3}{x^2-6x+9} + \frac{x-2}{3x-9} = \frac{x}{2x-6}$ -6, 5
- 48.** $\frac{3-2y}{y+1} - \frac{10}{y^2-1} = \frac{2y+3}{1-y}$ No solution

In Exercises 49–54, a rational function f is given. Find all values of a for which $f(a)$ is the indicated value.

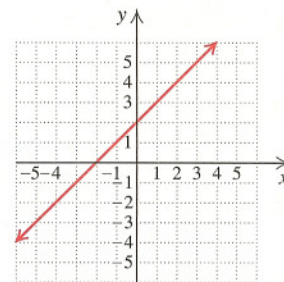
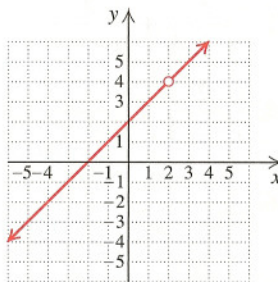
- 49.** $f(x) = 2x - \frac{15}{x}$; $f(a) = 7$ - $\frac{3}{2}$, 5
- 50.** $f(x) = 2x - \frac{6}{x}$; $f(a) = 1$ - $\frac{3}{2}$, 2

- 51.** $f(x) = \frac{x-5}{x+1}$; $f(a) = \frac{3}{5}$ 14
- 52.** $f(x) = \frac{x-3}{x+2}$; $f(a) = \frac{1}{5}$ $\frac{17}{4}$
- 53.** $f(x) = \frac{12}{x} - \frac{12}{2x}$; $f(a) = 8$ $\frac{3}{4}$
- 54.** $f(x) = \frac{6}{x} - \frac{6}{2x}$; $f(a) = 5$ $\frac{3}{5}$

For each pair of functions f and g , find all values of a for which $f(a) = g(a)$.

- 55.** $f(x) = \frac{x+1}{3} - 1$,
 $g(x) = \frac{x-1}{2} - 1$
- 56.** $f(x) = \frac{x+4}{3x}$,
 $g(x) = 2 - \frac{x+8}{5x}$ 2
- 57.** $f(x) = \frac{12}{x^2-6x+9}$,
 $g(x) = \frac{4}{x-3} + \frac{2x}{x-3}$ -3, 4
- 58.** $f(x) = \frac{14}{x^2-25}$,
 $g(x) = \frac{x}{x+5} - \frac{5}{x-5}$ -3, 13

TW 59. Below are unlabeled graphs of $f(x) = x + 2$ and $g(x) = (x^2 - 4)/(x - 2)$. How could you determine which graph represents f and which graph represents g ?



TW 60. Explain the difference between adding rational expressions and solving rational equations.